

Thin-Plate Spline Analysis of Mandibular Growth

Lorenzo Franchi, DDS, PhD^a; Tiziano Baccetti, DDS, PhD^a; James A. McNamara Jr, DDS, PhD^b

Abstract: The analysis of mandibular growth changes around the pubertal spurt in humans has several important implications for the diagnosis and orthopedic correction of skeletal disharmonies. The purpose of this study was to evaluate mandibular shape and size growth changes around the pubertal spurt in a longitudinal sample of subjects with normal occlusion by means of an appropriate morphometric technique (thin-plate spline analysis). Ten mandibular landmarks were identified on lateral cephalograms of 29 subjects at 6 different developmental phases. The 6 phases corresponded to 6 different maturational stages in cervical vertebrae during accelerative and decelerative phases of the pubertal growth curve of the mandible. Differences in shape between average mandibular configurations at the 6 developmental stages were visualized by means of thin-plate spline analysis and subjected to permutation test. Centroid size was used as the measure of the geometric size of each mandibular specimen. Differences in size at the 6 developmental phases were tested statistically. The results of graphical analysis indicated a statistically significant change in mandibular shape only for the growth interval from stage 3 to stage 4 in cervical vertebral maturation. Significant increases in centroid size were found at all developmental phases, with evidence of a prepubertal minimum and of a pubertal maximum. The existence of a pubertal peak in human mandibular growth, therefore, is confirmed by thin-plate spline analysis. Significant morphological changes in the mandible during the growth interval from stage 3 to stage 4 in cervical vertebral maturation may be described as an upward-forward direction of condylar growth determining an overall “shrinkage” of the mandibular configuration along the measurement of total mandibular length. This biological mechanism is particularly efficient in compensating for major increments in mandibular size at the adolescent spurt. (*Angle Orthod* 2001;71:83–92.)

Key Words: Morphometrics; Cephalometrics; Craniofacial growth; Mandible

INTRODUCTION

Craniofacial development and growth involve both size and shape variations. The anthropological and clinical significance of these changes is related to the assessment of growth potentials, the diagnosis of skeletal disharmonies, and the establishment of a proper orthopedic/orthodontic treatment plan.

The growth rate of craniofacial skeletal structures such as the mandible is not linear during development. In par-

ticular, classic studies have identified a pubertal spurt in mandibular growth, characterized by great individual variations in onset, duration, and rate.^{1–6} Mandibular skeletal maturity can be assessed by means of several biologic indicators: increase in body height,^{1,3} skeletal maturation of the hand and wrist,⁷ dental development and eruption,^{8,9} and menarche.^{10,11}

The evaluation of growth changes in the human mandible traditionally has been performed by means of cephalometric analyses of lateral radiographs of the craniofacial complex. The conventional metrical approach to the description of morphological forms, and conventional cephalometrics in particular, however, has proved to be insufficient for the analysis of size and shape changes of complex anatomical forms such as the human mandible. Lines and angles measured by traditional methods are not able to provide information about where the growth change has occurred.¹² The use of conventional cephalometrics is not coordinate free or invariant, but rather is dependent on the coordinate system.¹³

New descriptive methods of shape and shape changes have been developed and implemented as major improvements when compared with conventional cephalometrics.^{13–19}

^a Research Associate, Department of Orthodontics, The University of Florence, Florence, Italy; Thomas M. Graber Visiting Scholar, Department of Orthodontics and Pediatric Dentistry, School of Dentistry, The University of Michigan, Ann Arbor, Mich.

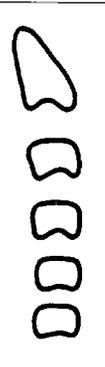
^b Thomas M. and Doris Graber Endowed Professor of Dentistry, Department of Orthodontics and Pediatric Dentistry, School of Dentistry; Professor of Cell and Developmental Biology, School of Medicine; and Research Scientist, Center for Human Growth and Development, The University of Michigan, Ann Arbor, Mich.

Corresponding author: Dr Lorenzo Franchi, Via G. F. Mariti 10, 50127 Firenze, Italy.
(e-mail: condax@tin.it)

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TABLE 1. Correspondence Between Skeletal Maturation Stage and Mean Chronological Age in the Examined Samples (from O'Reilly and Yanniello,¹¹-modified)

						
	Stages of cervical vertebral maturation					
	1	2	3	4	5	6
Age (mean ± SD)	9y 1m ± 1y 1m	10y 1m ± 1y 1m	11y 1m ± 1y 1m	12y 2m ± 1y 1m	13y 2m ± 1y 2m	14y 3m ± 1y 1m

Among these methods, Bookstein's innovations (tensor analysis, shape-coordinate analysis, thin-plate spline analysis) have been used to investigate modifications in shape related both to facial growth and to treatment.²⁰⁻²⁸ Mandibular shape and dimensions on lateral cephalograms have been investigated by different morphometric approaches. In particular, elliptic Fourier analysis of mandibular shape has been performed on the mandibular outlines digitized from the tracings of the Bolton standards from 1 to 18 years of age.²⁹ Finite element analysis has been used to investigate mandibular morphology in subjects with Class III malocclusion.³⁰ Tensor analysis,²⁴ shape-coordinate analysis,²⁷ and thin-plate spline analysis^{26,28} have been applied to the study of growth changes in the mandible of treated and untreated subjects with Class III malocclusion.

In the perspective of a comprehensive analysis of mandibular growth changes from infancy into adulthood, the above-mentioned studies show limitations because they either omit the evaluation of skeletal changes at the pubertal growth spurt^{24,27,28} or analyze cross-sectional samples.^{25,26,29,30} In addition, most of the studies deal with subjects affected by skeletal disharmonies.^{24-28,30}

The aim of this study is to apply an appropriate morphometric technique (thin-plate spline analysis) to the appraisal of mandibular shape or size growth changes around the pubertal spurt in a longitudinal sample of subjects with normal occlusion.

MATERIALS AND METHODS

Sample

The sample used in this study was composed of 29 subjects (15 men and 14 women) selected from the files of the

University of Michigan elementary and secondary school growth study.³¹ The use of archival radiographs conformed to institutional standards at the University of Michigan, because all human subjects had participated after providing informed consent to a protocol that had been reviewed and approved by an appropriate institutional board. All subjects presented with normal occlusion (Class I molar and canine relationships, normal overbite and overjet), with no vertical or sagittal skeletal discrepancies and with a well-balanced facial profile.

Lateral cephalograms at 6 different developmental phases (T1, T2, T3, T4, T5, and T6) were used for the analysis. All films were taken at a standardized subject-to-film distance so that the enlargement of each film was 12.92%. The 6 phases corresponded to 6 different maturational stages in cervical vertebrae according to the evaluation method of Lamparski.¹⁰ This procedure has proved to be effective and clinically reliable for the appraisal of skeletal maturation in growing subjects.^{11,32-34} The stages of cervical vertebral maturation are related to the mandibular growth changes that take place during puberty.³⁴ The 6 stages include observations before the peak (ie, during the accelerative growth phase—vertebral stages 1 to 3) and observations after the peak (ie, during the decelerative phase of growth—vertebral stages 4 to 6). Pubertal growth peak occurs on average between vertebral stage 3 and 4.³⁴ The correspondence between skeletal maturation stage and mean chronological age in the examined sample for the 6 developmental phases is reported in Table 1.

Each lateral cephalogram was traced on frosted acetate (0.03" or 0.762 mm thick) by 1 investigator (Dr Franchi) and checked by another investigator (Dr Baccetti). To in-

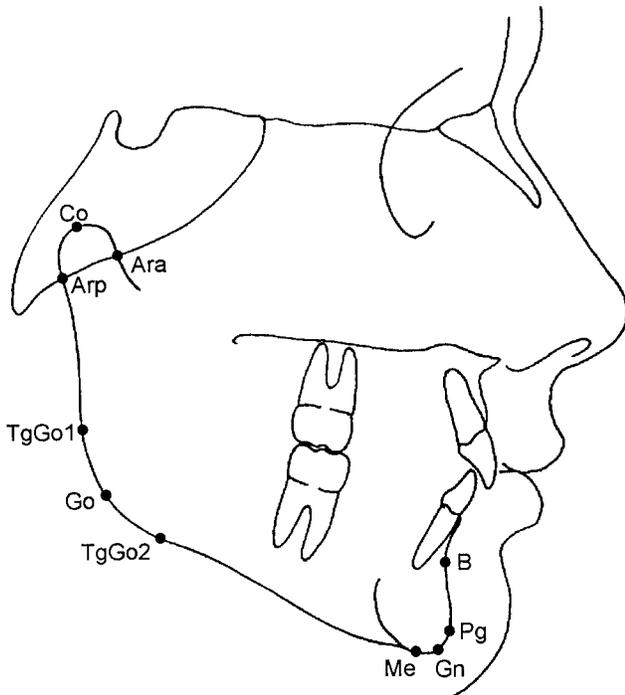


FIGURE 1. Ten mandibular landmarks used in this study superimposed on the cephalogram of a subject with normal occlusion.

crease the reliability of the landmarks selected, the cephalograms were taped to a light box of uniform brightness in a darkened room. A cross-wires cursor was used to digitize the landmarks. Ten mandibular landmarks were identified and digitized (Figure 1; Table 2) by means of appropriate software³⁵ (Viewbox, Version 2.0, D Halazonetis, Kifissia, Greece) and a digitizing table (Numonics, Lansdale, Pa). Method error in landmark identification is reported elsewhere.²⁸

Thin-plate spline analysis

Thin-plate spline (TPS) transformation produces a rigorous quantitative analysis of the spatial organization of shape change.³⁶ In TPS analysis, the differences in 2 configurations of landmarks are expressed as a continuous deformation by using regression functions in which homologous points are matched between forms to minimize the bending energy.³⁷ “Bending energy” can be defined as the energy that would be required to bend an infinitely thin metal plate over 1 set of landmarks so that the height over each landmark is equal to the coordinates of the homologous point in the other form.³⁸ TPS analysis facilitates the construction and display of transformation grids that capture the shape change between forms as an evolution of the method originally proposed by D’Arcy Thompson in 1917.³⁹ For a more detailed review of theoretical bases and calculation procedures of TPS morphometrics, see Bookstein,^{13,38} Rohlf and Marcus,⁴⁰ Rohlf et al,⁴¹ and Dryden and Mardia.⁴²

TABLE 2. Definitions of Landmarks Used in This Study

Abbreviations	Mandibular Landmarks
Ara	Articulare anterior (intersection of the anterior contour of the condyle and the posterior cranial base)
Co	Condylion (most posterior-superior point of the condyle)
Arp	Articulare posterior (intersection of the posterior contour of the condyle and the posterior cranial base)
TgGo1	Tangent gonion 1 (point of tangency of the line passing through Arp to the gonial region)
Go	Gonion (midpoint of the angle of the mandible)
TgGo2	Tangent gonion 2 (point of tangency of the line passing through Me to the gonial region)
Me	Menton (the most inferior point on the symphyseal outline)
Gn	Gnathion (the most anterior-inferior point on the contour of the bony chin symphysis)
Pg	Pogonion (the most anterior point on the contour of the bony chin)
B	B Point (the deepest point of the concavity on the anterior contour of the bony chin)

In this study, TPS software (Version 1.19, Ecology & Evolution, SUNY, Stonybrook, NY) computed the orthogonal least-squares Procrustes average configuration of mandibular landmarks in the examined subjects at T1 through T6, by using the generalized orthogonal least-squares procedures described in Rohlf and Slice.⁴³ The average mandibular configurations were subjected to TPS analysis by contrasting the average configurations at the 6 developmental phases (T2 vs T1, T3 vs T2, T4 vs T3, T5 vs T4, and T6 vs T5). Statistical analysis of shape differences was performed by means of permutation tests with 1000 random permutations on Wilk’s lambda statistics. Permutation tests were carried out because most landmarks slide along curves when shape changes are analyzed.

Centroid size was used as the measure of the geometric size of each mandibular specimen and was calculated as the square root of the sum of the squared distances from each landmark to the centroid of each specimen’s configuration of landmarks.¹³ Differences in size at the 6 developmental phases (T1 through T6) were tested by means of paired-samples *t*-tests ($P < .01$).

For those growth intervals showing significant shape differences, a test for allometry checking for shape depending on size was carried out. Statistical computations for centroid size analysis were performed with computer software (SPSS, Release 6.1.3, SPSS Inc, Chicago, Ill).

RESULTS

The results of the permutation tests for all the comparisons (T1 through T6) are shown in Table 3. Statistically significant differences between the landmark configurations of the mandible occurred only for the comparison T4 vs T3. Paired-samples *t*-tests revealed significant differences

TABLE 3. Procrustes Distances (d) Between Successive Average Shape Configurations (T1 through T6) and Results of Permutation Tests (p)

Growth Intervals	d	p
T1-T2	0.0065	0.104
T2-T3	0.0075	0.548
T3-T4	0.0140	0.001 ^a
T4-T5	0.0083	0.914
T5-T6	0.0101	0.085

^a Statistically significant.

TABLE 4. Results of Paired *t*-Tests for the Comparisons of Centroid Size at the 6 Different Developmental Stages

Developmental Stage	Mean	SD	<i>t</i> -Value	<i>p</i>
T1	1423.08	73.19	10.80	.000
T2	1456.45	70.99	9.91	.000
T3	1487.60	72.42	11.11	.000
T4	1531.15	78.03	8.69	.000
T5	1575.14	88.09	8.25	.000
T6	1609.09	85.05		

in centroid size for the comparisons at all developmental phases (Table 4).

The test for allometry for the comparison T4 vs T3 showed that significant shape changes were not significantly dependent on size differences ($F = 1.432$; $P = .175$).

TPS analysis allowed for graphical display of shape changes in the mandibular configuration at the 6 developmental phases. For each interval, the total warps are presented (Figure 2). As for the graphical displays related to growth intervals T1 to T2, T2 to T3, T4 to T5, and T5 to T6, no appreciable deformations of the transformation grids were recorded (Figures 2a, 2b, 2d, 2e).

The statistically significant shape change for the growth interval from T3 to T4 consisted of a compression in the horizontal axis in the region of the mandibular condyle (landmarks Ar and Co), leading to an overall shrinkage of the mandibular configuration along the measurement of total mandibular length (Co-Pg; Figure 2c).

DISCUSSION

Major advantages of TPS analysis applied to cephalometric landmark configurations with respect to both conventional cephalometrics and to previous morphometric techniques (tensor analysis, shape-coordinate analysis) include (1) an optimal superimposition of landmarks for the analysis of shape change in complex skeletal configurations without the use of any conventional reference line; (2) an explanatory visualization of the deformations caused by growth/treatment using transformation grids; and (3) the decomposition of generalized modifications into more specific, local changes. TPS analysis in this study was performed

to provide information about mandibular shape and size changes in relation to skeletal maturation in growing normal subjects. Specific features of this study were the following:

1. The longitudinal analysis of a sample of untreated subjects with normal occlusion and with well-balanced craniofacial skeletal relationships along a period spanning their whole adolescence.
2. The application of a reliable indicator of skeletal maturity (maturational stages in cervical vertebrae) to define time periods for sample evaluation.
3. The inclusion of the condyle among the examined mandibular structures for a more complete analysis of the mandibular shape.²⁹
4. The use of adequate statistical methods to evaluate shape and size changes at different stages and to quantify allometry of significant shape change.

Graphical display and statistical analysis of shape changes in the examined sample showed minor, nonsignificant modifications in mandibular morphology at all different maturational stages, with the exception of period T3 to T4 (Figure 2c; Table 3). This period corresponds to peak growth velocity in somatic and mandibular skeletal maturation (adolescent growth spurt).⁴⁴⁻⁵³ However, substantial and significant changes in mandibular size appear to take place at all examined growth intervals (Table 4). Hence the importance of a nonconventional biometric analysis, such as TPS, to appraise modifications in size independently from modifications in shape.²⁹ It should be noted that the mandible exhibited the highest absolute values for size change during the period T3 to T4 (43.54) and T4 to T5 (43.99). Moreover, most of the contributions in the literature identify a period of minimal velocity in somatic growth immediately before the onset of the adolescent growth spurt (prepubertal minimum).^{6,54,55} The findings of this study provide additional evidence in this regard. The analysis of centroid size of average mandibular configuration revealed the lowest value for size increment during the growth interval from T2 to T3 (31.15).

The biological interpretation of the morphometric findings is of interest. Significant morphological changes in the mandible during the growth interval from T3 to T4 may be described as an upward-forward direction of condylar growth determining an overall "shrinkage" of the mandibular configuration along the measurement of total mandibular length (Co-Pg; Figure 2c). This biological mechanism, defined as "anterior morphogenetic rotation" of the mandible,^{56,57} is able to dissipate excessive mandibular growth increments in relation to the maxilla, and it appears to be particularly efficient in compensating for major increments in mandibular size at the adolescent spurt.

Significant mandibular reshaping during the pubertal growth spurt occurred mainly in the condylar region. We therefore recommend including the condylar process among

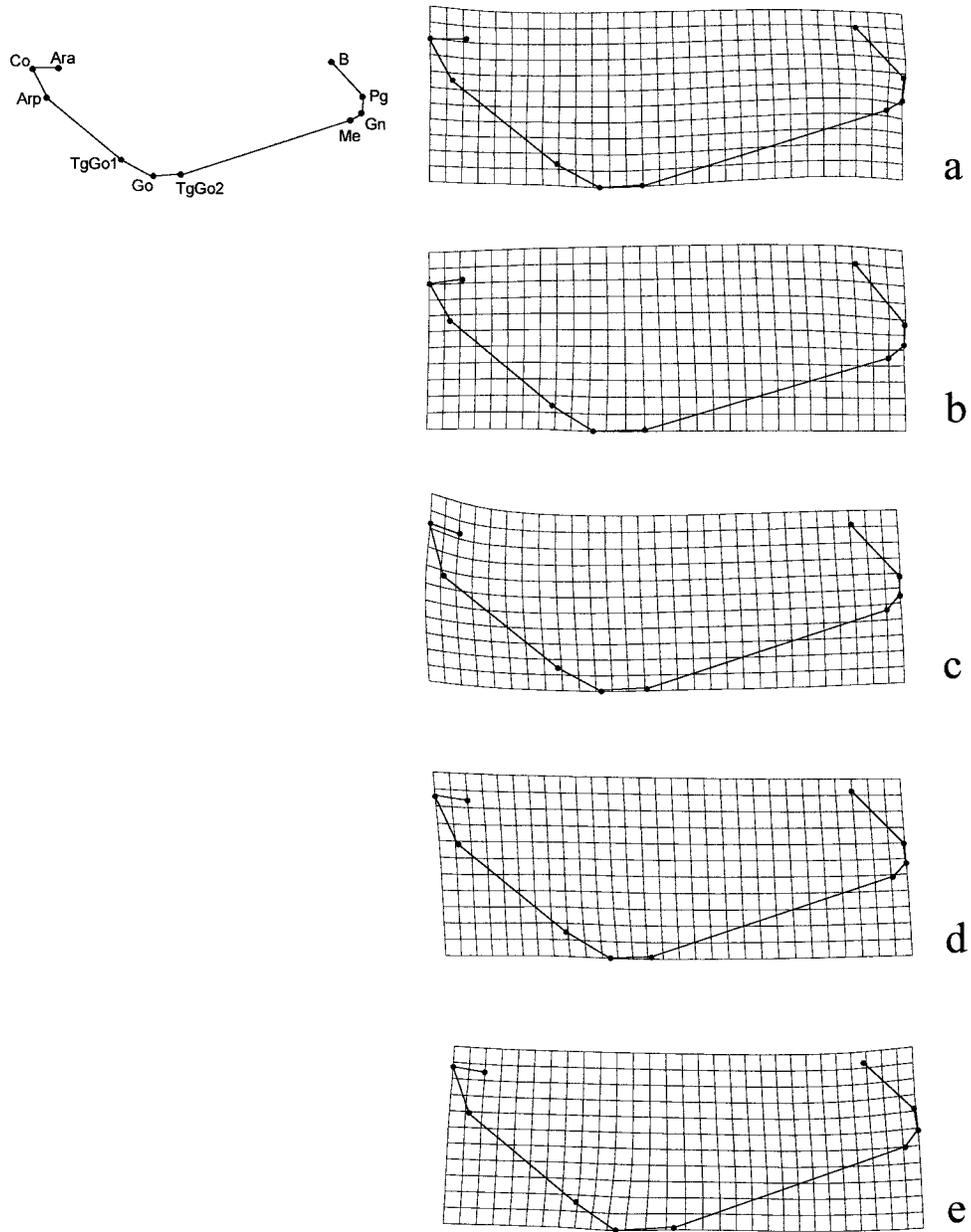


FIGURE 2. Thin-plate spline graphical display for the 5 growth intervals from T1 through T6: (a) T2 vs T1, (b) T3 vs T2, (c) T4 vs T3, (d) T5 vs T4, and (e) T6 vs T5. Magnification factor = 4 \times .

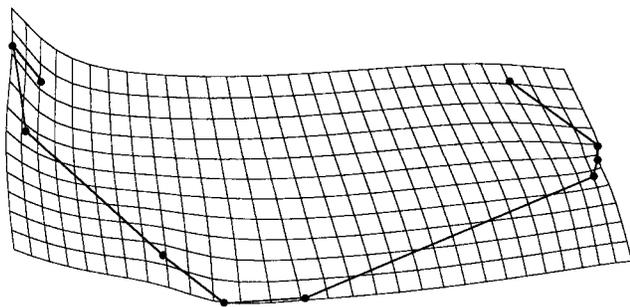


FIGURE 3. Thin-plate spline graphical display for overall growth interval (T6 vs T1). Magnification factor = 3 \times .

the mandibular structures investigated by means of morphometric analyses.²⁹ A few mandibular changes also can be detected at the level of the symphysis along with growth. In particular, a slight deformation in a forward direction at the antero-inferior contour of the symphysis has been assessed at different growth intervals (T1 to T2, T2 to T3; Figures 2a,b). This morphologic change probably should be ascribed to a remodeling process involving slight apposition at the antero-inferior border of the symphysis, as indicated in the classic cephalometric works by Björk² and Björk and Skieller.⁵⁸ The analysis of overall morphologic changes in the mandibles of normal subjects during the entire period

examined (T1 to T6; Figure 3) reveals a closure of the gonial angle associated with an upward-forward direction of growth at the condyle and with an upward-backward direction of growth at the symphysis, thus confirming the tendency to anterior morphogenetic rotation of the mandible.

The findings of this study indicate that significant modifications in shape of the mandible associated with the greatest increase in size take place between stages 3 and 4 in cervical vertebral maturation. The existence of a pubertal peak in human mandibular growth then is substantiated directly, and the reliability of the stage of cervical vertebral maturation as biologic indicator of mandibular skeletal maturity³⁴ is corroborated indirectly. It has been demonstrated that the effectiveness of functional or orthopedic treatment of mandibular deficiency in Class II skeletal disharmonies significantly depends on the biologic responsiveness of the condylar cartilage, which in turn greatly depends on mandibular growth rate.^{6,55} The results of this study show that major mandibular growth changes in correspondence of a specific stage of cervical vertebral maturation (stage 3 to 4) represent the most favorable period for the correction of mandibular deficiency, because it includes the ascending portion of the pubertal growth acceleration.

TPS analysis appears to be particularly efficient for the description and statistical evaluation of size and shape variations occurring during craniofacial growth and development. Further applications of this morphometric method in dentofacial orthopedics may consist of morphologic and dimensional comparisons between groups of treated and untreated individuals. Future improvements of the method will comprise the implementation of the analysis in 3 dimensions and the use of outlines of biological structures instead of landmark points through the combination of TPS with Procrustes statistics for the incorporation of outline information (edgewarp analysis).

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