# On Applications of Geometric Morphometrics to Studies of Ontogeny and Phylogeny

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The field of geometric morphometrics is relatively new (see Bookstein, 1991, Rohlf and Marcus, 1993) and has shown very rapid progress over the last few years. As might be expected during a period of rapid development, there can be technical problems in some of the pioneering studies as biologists attempt to apply the new tools. It is only now becoming clear how the new techniques should be combined in order to carry out comprehensive analyses of real datasets. Bookstein (1996b) gave a list of recommendations for such applications and Bookstein (1996a) gave several comprehensive examples of morphometric analyses. However, these accounts do not address some of the types of applications that are of particular interest in systematic biology. Bookstein (1994) pointed out problems with using geometric morphometric methods in the usual character-based cladistic studies. He emphasized that morphometrics cannot supply homologous shape characters. The purpose of this note is to comment some recent applications of morphometric methods in systematic biology.

This note is particularly concerned with the morphometric methods used by Zelditch et al. (1992, 1993, 1995), Swiderski (1993), Fink and Zelditch (1995), and Zelditch and Fink (1995). These methods were also used by Burke et al. (1996). For convenience, these studies will be referred to using the acronym Z&F. In these studies the authors investigated the use of partial warps (Bookstein, 1991) as variables in ontogenetic and taxonomic studies. They were impressed by their observation that differences in partial warps scores (Rohlf, 1993b) corresponded to shape differences that could be localized on the bodies of the organisms. They also found differences in these variables between developmental stages and between species. They concluded that partial warps could be interpreted and used as traditional taxonomic characters and would be useful in evolutionary studies.

Lynch et al. (1996) tried to interpret the partial warps they obtained in their study but they were cautious about using them in the ways advocated by Z&F. They suggested that extensive simulation studies needed to be done to validate the Z&F approach. Naylor (1996) investigated their approach using data based on a simulated phylogeny. Even though his simulated phylogeny was based on a sequence of simple morphological changes and had no homoplasy, the results showed high levels of homoplasy and did not recover the morphological changes used to create the phylogeny.

The present paper is concerned with theoretical problems rather than empirical problems requiring validation through simulations. The fundamental problem is that Z&F interpreted the partial warps as homologous and biologically meaningful variables—rather than as mathematically elegant but biologically arbitrary variables whose definition is not based on any covariance patterns in the data. There is also a problem in their choice of the so-called “reference.” This problem is discussed first as it provides the background that makes it easier to explain the other, more important, problems. These include ignoring the fact that partial warps are usually highly correlated, using methods whose results are very sensitive to different choices of the reference, and a warp by warp examination and interpretation of shape variables. Some suggestions are also made for the ways in which morphometric methods can be used to study ontogeny and phylogeny. The companion note by Adams and Rosenberg (1997) examines the Z&F protocol in more detail to demonstrate the consequences of different choices of a reference and different rotations of the tangent space.

## SHAPE SPACES AND THE CHOICE OF A REFERENCE

Landmark-based methods of geometric morphometrics are based on Kendall's (1981, 1984) definition of a shape space. Shapes (as captured by configurations of digitized landmarks) can be plotted as points in this multidimensional space. The distances between points in this space are invariant to variation in the location, orientation, and scale of the coordinate system in which the specimens are digitized. It is assumed that there are a fixed number of landmarks, $p$. While the geometry of Kendall's shape space is complicated, one can visualize it as the surface of a high-dimensional sphere. Thus, the appropriate distances between points are Procrustes distances, which are geodesic (great circle) distances usually measured in radians. By definition, similar shapes are those which are close together and dissimilar shapes are far apart with respect to this metric. The power of the geometric approach is that this shape space captures all possible variations in shape of configurations of landmarks. It also excludes information about variation in translation, rotation, and size (sometimes these are called nuisance parameters since they are not of interest as descriptors of shape variation).

Since Kendall's Shape space corresponds to the $kp-k-k(k-1)/2-1$ dimensional surface of a high-dimensional sphere it has a non-Euclidean geometry. As a result conventional linear multivariate statistical methods are not appropriate. However, when variation in shape is relatively small it is possible to approximate shape space by a linear space of the same dimension tangent to shape space. For example, in the case of shapes consisting of $p$ = 3 landmark points digitized in $k$ = 2 dimensions, one can visualize shape space as the surface of an ordinary sphere. The tangent space is then an Euclidean plane touching the surface of the sphere. Somewhat inappropriately, the tangent point is called the "reference" in geometric morphometrics. The statistical distribution of points in shape space is approximated by the distribution of the projections of the points from the surface of the sphere onto the tangent space (Fig. 1 shows a side view). Due to the foreshortening of the projections as one moves away from the reference (and shape space curves away from the tangent space), Euclidean distances between pairs of points in a tangent space are smaller than their corresponding Procrustes distances. This distortion becomes large as one considers points far from the reference. The approximation is best when the reference is close to the set of points being projected. Using an extreme point as a reference would clearly be undesirable. Bookstein (1996b: 145) recommended that one "… use a sensible reference form … best taken as the grand mean form" to optimize the approximation of shape space by the tangent space. For this reason, the reference should be taken as the mean of the observed shapes after differences due to location, orientation, and size are removed using generalized orthogonal least-squares Procrustes analysis (GLS, Rohlf and Slice, 1990).

Zelditch et al. (1992), in a study of cotton rats from 1 to 30 days old, used the mean 1-day-old form as the reference (which they called the "starting form"). Fink and Zelditch (1995), Zelditch and Fink (1995), and Zelditch et al. (1995), in studies of piranhas, used an average of three juveniles of an outgroup species as the reference. These choices were made because it seemed logical to compare different developmental stages to an initial one or to compare taxa to an outgroup. However, as discussed above, the reference in geometric morphometrics is not a standard to which other shapes are to be compared. The reference is simply the shape that defines the tangent space approximation so that linear statistical methods can be used to study shape. The choice of reference has nothing to do with the statistical comparisons to be made within the tangent space. To avoid this confusion about the purpose of the reference, Rohlf et al. (1996) referred to the reference as the “tangent point” or the “tangent configuration.” Once a tangent space is defined, one has a set of shape variables that can be used to make any desired comparisons. Developmental stages can be compared to an early stage or taxa to outgroups etc. using standard statistical designs and multivariate methods.

A program, tpsSmall (Rohlf, 1997), is available to help one assess the accuracy of the approximation of shape space by the tangent space. It measures the accuracy by the correlation between Euclidean distances in the tangent space and Procrustes distances in shape space. Uncentered correlations are computed by the program since any line corresponding to the relationship between these distances must go through the origin. The approximation is usually very good and little nonlinearity is visible.

The effects of different choices of a reference on distances in the tangent space are expected to be relatively minor. The well known rat growth data (data from Bookstein, 1991) can be used as an example. It consists of eight well-spaced landmarks digitized around the brain case of 164 rats from 7 to 150 days of age. The effects of several different choices for the reference were investigated (the overall mean, the mean of the 7-day-old rats, and each of the first three 7-day-old specimens). The entire set of 164 specimens was projected onto tangent spaces defined by the different references by using matrices of partial-warp scores based on each reference. These matrices were then used to compute matrices of distances among the 164 specimens. Uncentered correlations among the distance matrices for the three specimens and between each of them and the matrix based on the mean of the 7-day-old rats ranged from 0.99978 to 0.99998. The correlation between the distance matrix using the mean of the 7-day-old rats as a reference and the distance matrix using the mean of the entire data set as a reference was 0.99809. The partial warps ignore uniform shape differences (affine differences) which are very large in this dataset. If Bookstein’s (1996c) estimates of the uniform component are appended as additional variables then the correlation rises to 0.99986. Thus the patterns of similarities and differences captured by the tangent space are quite stable, i.e., the relative distances between the points in the tangent space are not changed much due to changes in the reference. Even though it may make little difference, the GLS consensus should be used as the reference since it is easy to compute and it minimizes errors in the approximation of shape space. On the other hand, the individual partial warps themselves are not very stable. This problem is discussed in the next section.

## SHAPE VARIABLES

When one uses conventional statistical analyses to study variation, to compare shapes, or to explore the covariation of shape with extrinsic variables, one needs shape variables. Shape variables can be generated in many different ways. The choice is important since different sets of variables have different interpretations and may lead to different statistical conclusions in some kinds of analyses. A simple set of shape variables is the set of $kp$ coordinates of the $p$ landmarks in $k$ dimensions for $n$ specimens after they have been aligned to the reference using GLS superimposition. The coordinate can be interpreted as variables that, when taken together, define the position of each specimen in tangent space. However, these variables are redundant since tangent space has only $kp-k-k(k-1)/2-1$ dimensions (after the removal of the effects of differences in translation, rotation, and size). One can obtain a non-redundant set of shape variables by using Bookstein shape coordinates (Bookstein, 1991) to align the specimens along one or more baselines. The resulting shape variables are easy to understand but they do not yield as good of an approximation of shape space as do the residuals from a GLS superimposition.

Another method (the one used by Z&F) is to use the reference to define $p-k-1$ principal warps and then separately project the $x$, $y$ (and possibly $z$) coordinates of the aligned specimens onto these vectors. The resulting variables are called partial warps. They are a set of shape variables that span the part of tangent space that corresponds to shapes that can be described in terms of non-linear deformations (localized changes) of the reference configuration. The partial warps are often called the nonaffine shape components. One can also generate shape variables that describe shapes that correspond to affine (i.e., nonlocal) shape changes in the reference (see Bookstein, 1996c). These are usually referred to as the uniform shape component and can be thought of as the $0^{th}$ partial warps.

Partial warps have the elegant interpretation of giving a coordinate system for the tangent space that also corresponds to a geometrical decomposition of the potential modes of shape variation into geometrically orthogonal components at different spatial scales. Partial warps with large eigenvalues (large bending energies) correspond to small-scale shape changes and warps with small eigenvalues correspond to large-scale shape changes. Note that the bending energies and the shape changes corresponding to each warp are based only on the configuration of landmarks in the reference (Bookstein, 1991). No information on shape variation or covariation is taken into consideration (Rohlf, 1993b). The partial warp scores for a specimen are scalar values for each dimension indicating how much of each principal warp is needed to account for the observed differences between the specimen and the reference. The specimens have no influence on the kind of shape change corresponding to each principal warp and hence have no influences on the partial warps.

While the tangent space is quite stable (as discussed above), the orientations and hence the interpretations of the partial warp axes are not very stable. One way to demonstrate this is to compute angles between principal warps based on different references. The angles can be computed as the absolute value of the arccosines of the product of one matrix of normalized principal warps times the transpose of another such matrix. Ideally, the result would be a matrix with angles of 0° down the diagonal and angles of 90° in the off-diagonal entries. The entries down the diagonal are shown in Table 1 for the rat data. The angles for comparisons of the first three specimens with the average of the 7-day-old rats are given in the first three rows. The angles varied from 24.8° to 87.8°. The comparison of the principal warps using the average of the 7-day-old rats as a reference versus using the mean of the entire dataset as a reference are given in the last row. The angles varied from 64.5° to 89.0°. There was also high variability in the off-diagonal angles in each of the comparisons. Thus the pattern of shape changes implied by each principal warp (and hence the biological interpretation of each partial warp) seem to be very sensitive to the choice for the reference.

Zelditch et al. (1995) emphasize that the partial warps corresponded to shape differences that can be localized on the body of the organisms. This is not, however, a unique property of partial warps. The entire space spanned by the partial warps (other than the $0^{th}$) as well as *all* linear combinations of the partial warps have this property. As Zelditch et al. (1992: 1169) pointed out, "There is no biological information … in the coefficients for each landmark in these eigenvectors. Rather, these principal warps provide a basis for comparison, a list of features, each progressively more localized, for comparisons of differences between forms." This statement also applies to the partial warps because the partial warps are just the principal warps multiplied by scalar weights for each dimension. One expects the space spanned by the partial warps to contain useful and biologically interpretable information—they span the part of tangent space that contains information on all possible shape variables that can be localized. There is, however, no reason to expect the partial warps to represent directions that should be especially interpretable biologically. Even though the landmarks may be homologous, it is difficult to think arbitrary linear combinations as being homologous. Curiously, Zelditch et al. (1992) described and interpreted each partial warp and performed tests of significance for age differences for each partial warp separately.

## BIOLOGICAL INTERPRETATION OF PARTIAL WARPS?

An important problem with the Z&F studies is that they use and interpret each partial warp separately. Zelditch and Fink (1995: 343) correctly observed that a limitation of the interpretation of Bookstein shape coordinates for studies of developmental integration is that "… we cannot know in advance of analysis which triangles span developmentally autonomous units and which combinations of triangles extend over developmentally integrated areas." Partial warps have the same kind of limitation since they are simply weighted principal warps whose pattern of deformation is determined in advance by the configuration of landmarks in the reference and not by the patterns of covariation in the data.

Zelditch et al. (1992: 1176) reported a higher level of integration of the different parts of the skull than they expected based on conventional interpretations of mammalian skull growth. They also reported a lack of evidence for developmental units corresponding to the facial skull or cranial base. Their conclusions were based on their observation that the large-scale partial warps described shape deformations across many different parts of the skull and that the effects of the small-scale partial warps were not localized corresponding to the traditional developmental units. These conclusions are unwarranted. The deformations corresponding to the principal warps (and hence the partial warps) are, as discussed above, functions of just the configuration of landmarks in the reference and does not take into consideration patterns of covariation among the landmarks. The fact that a partial warp does or does not align with biologically meaningful structures is fortuitous.

The following artificial example may be helpful. Fig. 2 shows a configuration with three landmarks in each of two groups. Using this configuration as a reference, the principal warps can be computed (they are shown separately as *x* and *y* deformations). Fig. 3 shows a configuration of landmarks in which the upper group of landmarks has been expanded to represent a simple change in a single biological feature. This change can also be shown as a thin-plate spline deformation of the reference as shown in Fig. 3b. Fig. 4 shows the decomposition of the deformation in Fig. 3b into its three partial warps. The *x* and *y* contributions to each partial warp are shown jointly. Partial warp 1 is principal warp 1 with a weight of 0.0 for the *x*-axis and –0.46 for the y-axis. Since the coefficient is negative for the *y*-axis, the implied deformation is a relative expansion of the sizes of both groups of landmarks. The second partial warp is principal warp 2 with a weight of 0.0 for the *x*-axis and 0.58 for the *y*-axis. This expansion of the upper group of landmarks relative to the lower group is the type of pattern that one would intuitively expect. However, it is too extreme in its compression of the lower region. The third partial warp is the third principal warp with weights of 1.0 for the *x*-axis and 0.0 for the *y*-axis. This warp also shows an extreme compression of the lower group of landmarks but this time in the *x* direction. The second and third partial warps are extreme in order to compensate for the pattern in the first partial warp that shows an expansion in the lower group of landmarks. Note that in a sense the effects of the partial warps are not really localized—their effects are best described as contrasts in which the expansions of some regions is relative to compressions in others. The partial warps in Figure 4 might seem to suggest integration since the warps have a complementary effect on the two regions. This is an unwarranted conclusion because the partial warps cannot show any pattern of deformation that is not given by the principal warps and the principal warps are only a function of the reference not of the covariance among the landmarks. Separate warp-by-warp examinations of the partial warps cannot reveal either a pattern of partitioning of landmarks into covarying groups nor their integration across groups.

Note that three partial warps were required to yield the simple morphological difference was used to create the configuration in Fig. 3a. Since the warps are based only on the reference it is unlikely that one of them would correspond to an appropriate morphological change. Similar results were found by Naylor (1996) in his simulations. In view of these properties of partial warps, it does not make much sense to try to interpret them as homologous characters. Bookstein (1994) discuses several additional theoretical reasons why partial warps should not be treated as homologous characters. However, when taken together the partial warps define a space that captures the variation in all possible non-affine shape variables. Paradoxically, it does this well even though each variable by itself is somewhat arbitrary and need not be especially interesting biologically.

## UNIVARIATE VERSUS MULTIVARIATE ANALYSES OF SHAPE

Another important problem with the methods used by Z&F is that they perform statistical analyses on the partial warps one at a time. Although the partial warps are geometrically orthogonal, they are not statistically orthogonal, i.e., they are not statistically independent. In fact, they tend to be highly correlated. For the rat growth data mentioned above, almost half of the correlations between pairs of partial warps were greater than 0.5 and many were larger than 0.8. Performing a series of univariate tests assumes either that the variables are statistically independent or that each variable is of particular biological interest a priori. Partial warps meet neither of these conditions. Standard multivariate statistical methods can be used to take these correlations into account and to make use of the tangent space as a whole.

It is essential that methods be used whose results do not depend upon the particular orientation of the partial warp axes in the tangent space. Thus Bookstein (1996b: 146) warns that one should not "… believe any multivariate analysis unless its substantive import is independent of the choice of this basis [i.e., the choice of shape variables] … Any finding that requires the use of partial warps is erroneous." What this means is that studies that analyze partial warps must use statistical methods whose results are invariant to the effects of orthonormal transformations of the variables. An orthonormal transformation is a linear transformation of a multivariate space that can be visualized as a rigid rotation of the coordinate axes. An important property of such transformations is that distances between pairs of points and angles between vectors are not effected by the transformation (Reyment and Jöreskog, 1993:46). Note that we are referring here to transformations of the multivariate space of shape variables and not to rotations of the organism. The principal warps are invariant to changes in the orientation of the organism. Rotations of the axes generate new shape variables. While individually these may or may not be more interpretable than the original axes, taken as a whole they capture the same information about shape variation and should lead to the same statistical results. This allows one to use the partial warps as shape variables without assuming them to be especially meaningful one at a time.

The protocols used by Z&F do not share the invariance properties described above. The most important problem is that they examined each principal warp individually to determine its usefulness as a taxonomic character. Ontogenetic changes were estimated for each warp by performing multiple regression analyses to predict size as a function of the *x* and *y*-components of each partial warp (a multivariate regression to predict shape as a function of size would have been more reasonable). When significant relationships were found, each partial warp was regressed on size. The results of these regression analyses were reduced to discrete states so they could be analyzed by parsimony analyses. This was done as a function of the existence of statistically significant regression coefficients and their signs. Although their effects may be small, the breaking up of continuous variables (regression slopes) is not invariant to rotations of axes nor is parsimony analysis based on the Manhattan metric.

On the other hand, the results of analyses such as principal components analysis (PCA) and clustering of Euclidean distance matrices are invariant to rigid rotations of axes. Methods such as multivariate analysis of variance (MANOVA), Hotelling's generalized $T^2$ tests, canonical variates analysis (CVA), multivariate regression, and Mahalanobis distances are invariant not only to rigid rotations but also to affine transformations as well (they are, for example, also invariant to the effects of multiplication of the axes by scale factors). Rohlf et al. (1996) give a number of examples of the application of these methods to geometric morphometrics.

Multivariate analyses can also be used to construct empirically interesting shape variables. For example, one can construct statistically orthogonal sets of variables that account for the majority of the variance in a few dimensions (principal components) or that best distinguish between two groups (discriminant functions). There are a number of ways to display such shape variables. A visually appealing method is to construct hypothetical shapes using the thin-plate spline. Rohlf et al. (1996) shows several examples. Computer programs are available from the Stony Brook Morphometrics www pages (`http://life.bio.sunysb.edu/morph`) that provide these visualizations for a variety of statistical analyses.

## DISCUSSION

Bookstein (1994) pointed out a number of theoretical problems in applying morphometric methods in character-based cladistic analyses. These problems are consequences of the fact that shape space is curved and high dimensional. While his arguments are somewhat abstract, the problems are real and cannot be ignored. Two problems are of particular interest and are discussed below.

First, due to the fact that Kendall's (1981, 1984) shape space is curved, the meaning of a shape change resulting from a specified change in the value of a shape variable differs depending upon the values of the other shape variables. That is, the deformation that results from changing shape variable *A* will not be exactly the same at different values of another shape variable *B.* Thus a change in shape variable *A* from 1 to 2 followed by a change in variable *B* from 0 to 1 does not yield the exact same final shape that would result if the change was in variable *B* followed by the change in variable *A*. Bookstein (1994) discussed this lack of commutivity of shape transformations. He also discussed a related problem that he refered to as the nonexistence of rectangles in shape space. These properties of shape variables are incompatible with methods that treat variables and their states as entities that can be manipulated using methods based on combinatorial mathematics. Of course, the magnitude of the effects of the curvature of shape space depends upon the magnitude of shape change. For small variation in shape and an appropriate choice of the reference, distances in shape space can be approximated quite well by Euclidean distances in the linear tangent space.

A second problem Bookstein (1994) referred to as the shape nonmonotonicity theorem. There are an infinite number of different arbitrary rotations of shape space. Each rotation leads to shape variables that can have different ordering and spacing of points along particular axes. For 3 non-collinear points in 2D or 4 non-collinear points in 3D one can easily find a shape variable that gives any ordering of the OTUs that one wishes. For more landmarks there are geometric constraints (*e. g.*, interior landmarks cannot be at either end of the orderings) but a great many orderings are still possible.

Thus, as discussed above, methods need to take into account the fact partial warps need not be of particular biological interest and that variables generated by different rotations of the tangent space may be of equal or greater interest. While Z&F were successful in finding taxonomically useful characters that does not mean that the partial warps they used are especially meaningful. Other linear combinations of the partial warps (*e. g.*, rotations resulting from slightly different choices of the reference configuration) are just as likely to be interpretable. Of course, one may find taxonomically useful characters serendipitously. All directions in tangent space correspond to shape variables and it is likely that they can be interpreted in some way. In order to use shape variables with a protocol that is not invariant to rotation one needs to justify that the selected variables are each of particular biological interest in comparison to an infinite number of other possible shape variables. Variables based on empirical patterns of covariation in the data are more likely to be biologically meaningful than those generated by an arbitrary a priori rotation of shape space.

An analogy to the use of Fourier analysis to study variation in outline shape may be helpful for those already familiar with that approach. Fourier coefficients give the weights for the contribution of the sine and cosine terms for each harmonic just as partial warp scores give the weights for each principal warp. Given a sufficient number of harmonics and carefully aligned outlines, Fourier coefficients can be used to define a space in which points correspond to shapes of entire outlines (Rohlf, 1990, 1993a) just as partial warp scores can be used to define a shape space where points correspond to landmark configurations. The different harmonics refer to different spatial scales as do the principal warps. One should not try to give biological interpretations to the individual harmonics (Bookstein et al., 1982; Rohlf, 1993a) and the individual partial warps should not be interpreted for similar reasons. The relationship between Fourier coefficients and the physics of vibrating strings is irrelevant to their use for the analysis of outline shape. Similarly, the relationship between partial warp scores and the physics of deformations of thin metal plates is irrelevant to their use for the analysis of the shapes of configurations of landmarks.

The comments expressed above should *not* be interpreted as implying that geometric morphometric methods cannot be used in evolutionary biology. Analyses must take into account that partial warps are continuous variables and the orientations of the partial warp axes are arbitrary biologically. Bookstein’s (1994) critique is mostly aimed at attempts to use partial warps as characters in cladistics and on the use of statistical distance coefficients to measure dissimilarity between shapes. It should also be clear that the present paper is concerned with general mathematical and statistical issues that have nothing to do with the phenetics/cladistics controversies since we are not concerned here with methods for creating classifications.

For phylogenetic estimation one could use Felsenstein’s (1981) maximum likelihood method for continuous data or Maddison’s (1991) squared-change parsimony methods since they are invariant to the effects of rotation. The maximum likelihood method is particularly appropriate since the Procrustes metric used to define shape space is consistent with modeling morphological change as a multidimensional random walk. Methods based on Manhattan distances or those that require the reduction of continuous variables into discrete states are not appropriate for geometric morphometric variables since their results are not invariant to the effects of arbitrary rotations of shape space. A limitation to the use of shape space to estimate phylogeny is the fact that morphometric analyses typically are limited to a single structure whose components are often highly correlated. Thus shape space often represents just a few independent characters (however defined). It will probably prove to be much more useful to fit an estimated phylogeny obtained from other data and study how shapes change along the estimated lineages (see Bookstein, 1994: 211).

The field of geometric morphometrics has reached a level of maturity where problems in ecophenotypy and evolutionary biology can now be investigated. The studies must be based, however, on a firm understanding of the methods being used to minimize the possibility of biases and statistical artifacts in the results. The new methods are very powerful and must be used with care.

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*Accepted:*

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Table 1. Angles (in degrees) between principal warps based on different references for the rat data from Bookstein (1991).

| | Principal warps | | | | |
|---|---|---|---|---|---|
| Comparisons | 1 | 2 | 3 | 4 | 5 |
| Specimen 1 vs. mean 7 day old | 33.7 | 63.0 | 69.6 | 74.0 | 87.8 |
| Specimen 2 vs. mean 7 day old | 38.6 | 39.0 | 39.8 | 44.5 | 54.8 |
| Specimen 3 vs. mean 7 day old | 73.1 | 45.9 | 24.8 | 51.7 | 84.1 |
| Mean 7 day-old vs. overall mean | 89.0 | 87.9 | 70.1 | 64.5 | 72.4 |

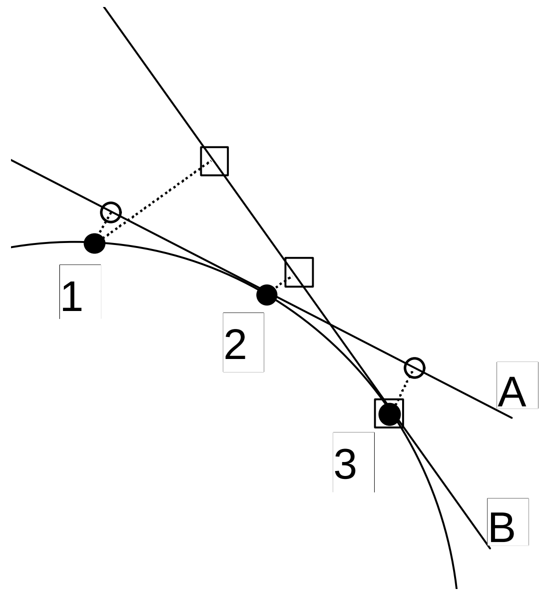


Figure 1. Projections of three points (●) in shape space onto two possible tangent lines. Tangent line A through the central point gives a less distorted representation of the relative positions of the projected points (○) than does tangent line B. On tangent line B, the projections (□) of points 1 and 2 are slightly closer together than the projections of points 2 and 3.

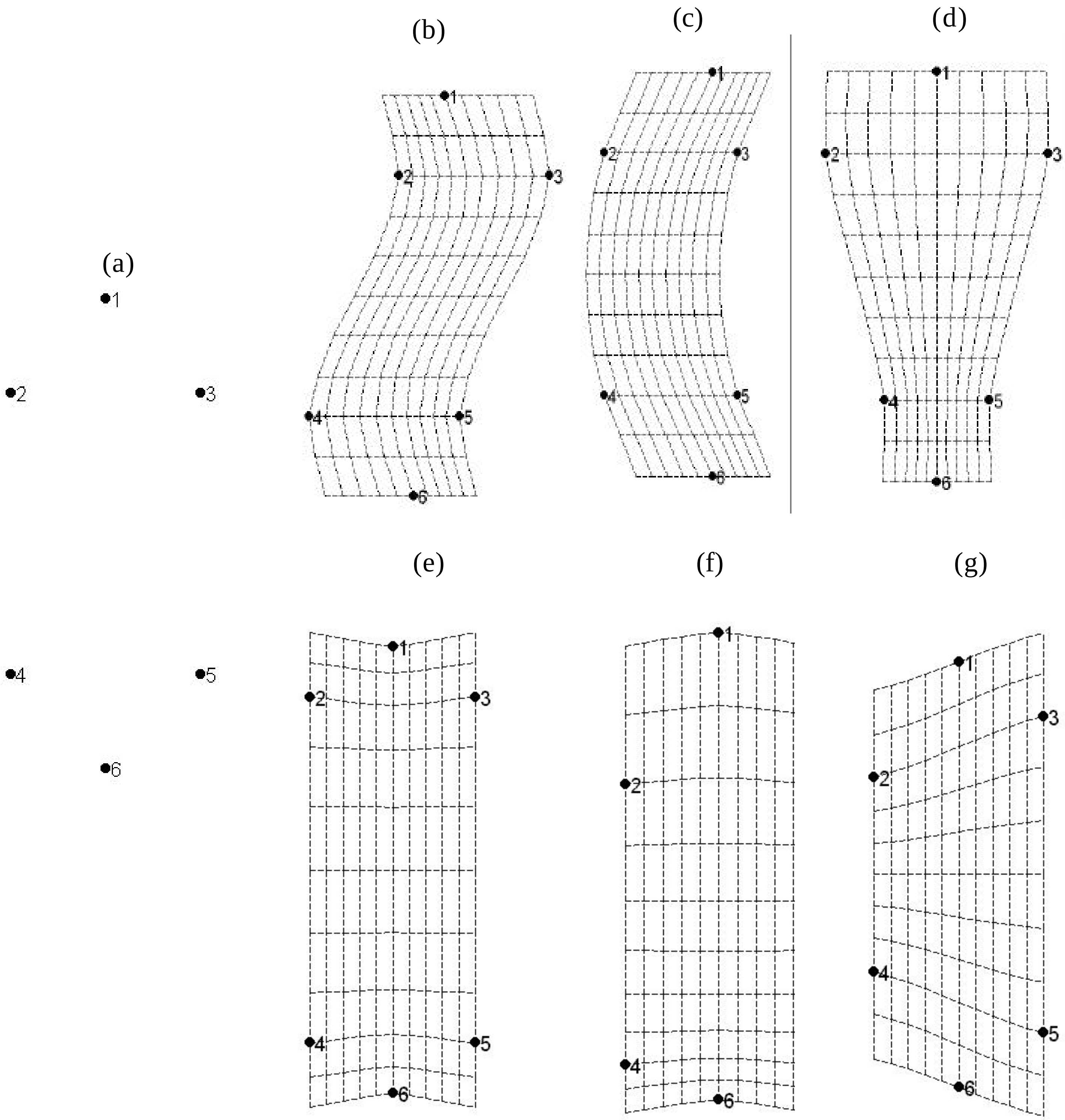
(a)
(b)
(c)
(d)
(e)
(f)
(g)

Figure 2. A reference configuration (a) and its principal warps (b-g). The warps are ordered from smallest (towards the left) to largest (right) spatial scale and expressed as deformation in x-coordinates (b-d) and in y-coordinates (e-g). The bending energies are 0.255, 0.140, and 0.125.

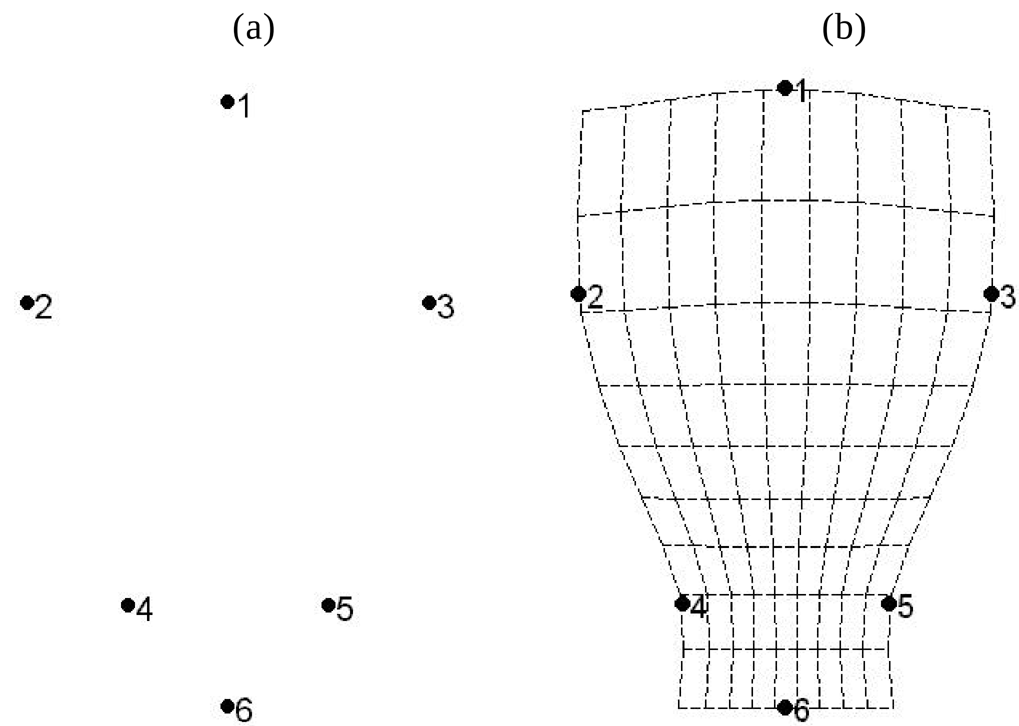


Figure 3. (a) Data configuration in which the size of the region delimited by landmarks 1—3 has been expanded (a). (b) Thin-plate spline from the reference in Fig. 2a to the data configuration.

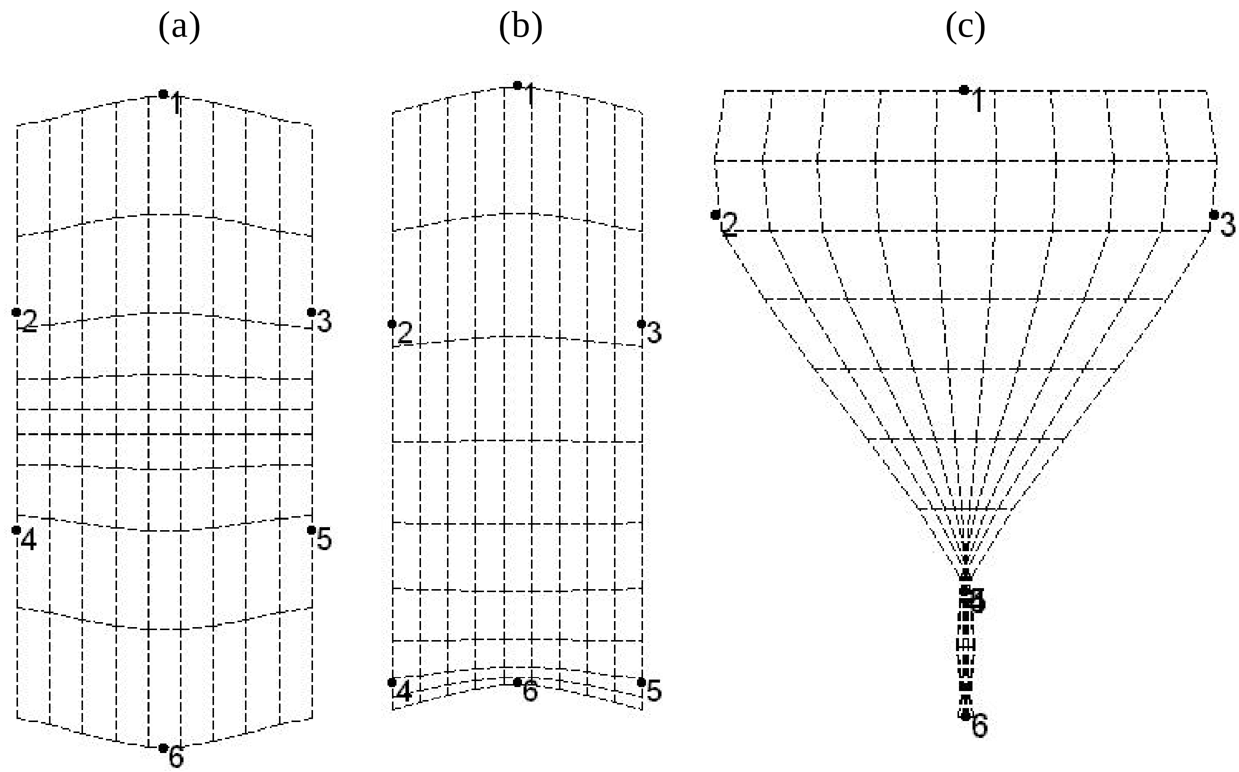


Figure 4. Partial warps for the specimen in Fig. 3a using the principal warps shown in Fig. 2. Despite the fact that the only change was the enlargement of a single region, three partial warps were required to express the change.

## Figure captions

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